Nonlinear analysis of piles in rock

Analyse non-linéaire des pieux en roche

J. M. AMIR, Consulting Engineer, Tel-Aviv Israel

SYNOPSIS

The behavior of shear piles in rock is analyzed by the spring model method, assuming an exponential relationship between sidewall shear and displacement. The resulting nonlinear differential equation, in terms of dimensionless force, may be solved by iterative finite differences. The load-settlement curves and axial force distribution obtained from this solution show good agreement with field measurements.

INTRODUCTION

Since the early seventies, drilled cast-in-situ piles have become the most important foundation method in the rocky regions of Israel. The main reason for this is that piling, especially in jointed and karstic rock, has obvious economical advantages over shallow footings (Amir 1983). The rapid advance in construction techniques has, however, left analytical techniques behind. In order to achieve safer and more economical design, engineers need better understanding of the way these piles function. The paper presented here proposes a new analytical approach in this direction.

SIDEWALL SHEAR MODELLING

Piles in rock (with the exception of short, large-diameter sockets) derive their capacity mainly from sidewall shear. The problem, therefore, is reduced to modelling the behavior of the pile in shear. Basically, there are three analytical techniques which can be used to model the load-deformation behavior of piles:

a) The spring model (Scott 1981)
b) The half-space model (Mattes & Poulos 1969)
c) The finite element method (Pells & Turner 1979).

The spring model was chosen for this work because of its relative simplicity, and because the extra computational effort involved with the other (more accurate) methods can only be justified if reliable material parameters are available. Unfortunately, rock testing techniques have not yet matured to this stage.

In its simplest form, the spring model is linear, and may be characterized by a single spring element. Although it can represent the behavior of piles under relatively small loads, this single-parameter model can simulate neither work-hardening nor yield.

The elasto-plastic model, which consists of a spring and a friction element connected in series, does represent yield phenomena. Still, it lacks continuity and is grossly in error in the working range of the pile, which is of most interest to the engineer.

Multi-element elasto-plastic models, consisting of a series of springs and friction elements, have none of the above shortcomings. Generally, the stress-displacement curves for such models are in the shape of broken lines. In the special case where both spring constants and friction values decrease in a geometrical progression (Fig. 1), the slopes of the successive sections also form a descending geometrical progression. In the limit, if an infinite number of elements is taken, a smooth curve, obeying Eq. (1), results:

$$\ln(\tau') = a - n.s \qquad (1)$$

where τ is the sidewall shear stress, s the displacement and both a and n are constants. Rearranging and integrating, Eq. (1) becomes:

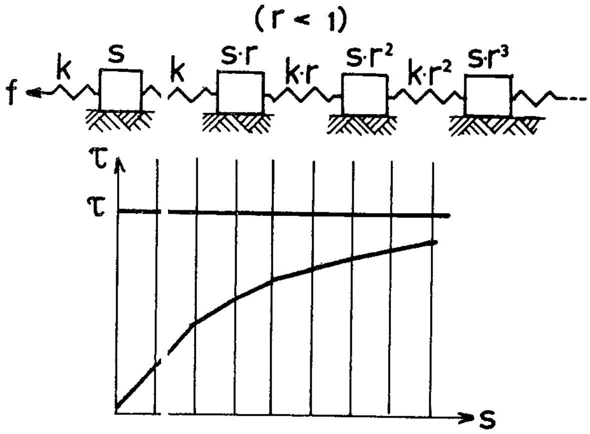


Fig. 1 - Multi-element elasto-plastic model

$$\tau = K - \frac{e^a}{n} e^{-ns} \tag{2}$$

Since $\tau = 0$ when $s = 0$, the integration constant K is equal to e^a/n, therefore:

$$\tau = m(1-e^{-ns}) \tag{3}$$

where m, the yield shear stress, is equal to $e^{a/n}$. This exponential expression has the required attributes of strain-hardening, continuity and eventual yield, still using only two parameters. Eq. (3) is identical to the expression suggested by Scott (1981), and to the load-settlement function for complete piles given by van der Veen (1953). By differentiating at the origin, it can be verified that mn is the corresponding tangent modulus.

For the linearly elastic case, it can be shown (Appendix I), that the pile modulus (the ratio of mean sidewall shear to pile settlement) is practically independent of the pile diameter. It was therefore assumed that, for given site conditions, the spring parameters m and n are constant.

Unfortunately, there is little published data regarding shear tests between concrete and rock. Results of such tests, reported by Pells et al. (1980) and Williams & Pells (1981), show that the exponential dependence of shear stress on displacement is basically correct.

LOAD-SETTLEMENT BEHAVIOR

Under an applied axial load Q, the pile deforms with depth z according to the function $s = s(z)$. For a pile section with a length Δl and circumference C, the axial force changes from F to $F + \Delta F$, and the equilibrium condition gives:

$$F' = \frac{dF}{dz} = -\tau \cdot C \tag{4}$$

substituting the value of τ from Eq. (3), one gets:

$$F' = -C.\ m(1 - e^{-ns}) \tag{5}$$

differentiating,

$$F'' = -C\ m\ n\ e^{-ns}.s' \tag{6}$$

Hooke's law for the same pile section yields:

$$s' = \frac{ds}{dz} = \frac{-F}{E\ A} \tag{7}$$

where E and A are the modulus of elasticity and the cross-sectional area of the pile, respectively. Substituting in (6):

$$F'' = C\ m\ n\ e^{-ns}.\ \frac{F}{E\ A} \tag{8}$$

Combining (5) and (8) yields:

$$F'' + \frac{n}{E.A} FF' - \frac{n.C.m}{E.A} F = 0 \tag{9}$$

Equation (9) may be normalized by the following substitutions:

$$\emptyset = F/Q \tag{10}$$

and

$$\zeta = \lambda \cdot z \tag{11}$$

so that Eq.(9) becomes:

$$\emptyset'' + \frac{n\ Q}{\lambda\ E\ A} \emptyset.\emptyset' - \frac{C\ m\ n}{\lambda^2\ E\ A} \emptyset = 0 \tag{12}$$

the derivatives being with respect to the dimensionless depth ζ. To simplify Eq. (12), it is convenient to make the following substitutions:

$$\lambda = \frac{n.Q}{E.A} \tag{13}$$

and

$$k = \frac{m.C.E.A}{Q^2.n} \tag{14}$$

thus:

$$\emptyset'' + \emptyset.\emptyset' - k.\emptyset = 0 \tag{15}$$

Eq. (15) is a non-linear differential equation in terms of the dimensionless force Ø. This is a convenient form, since in practice the boundary values are also known in terms of force: At the top ($\zeta = 0$) F=Q, so Ø is equal to unity. As for the bottom ($\zeta = \lambda l$), it was demonstrated (Mattes & Poulos 1969) that for normal slenderness ratios the proportion of the force reaching the bottom does not exceed a few percent. For shear piles in rock, in which debris is allowed to collect at the bottom, assuming zero force (Ø = 0) at the end will usually be quite appropriate. For extremely long piles, it may be more accurate to assume that the point of zero axial force is higher up, but this does not introduce any additional difficulty.

Eq. (15) has no closed-form analytical solution, and a conventional finite difference formulation produces non-linear algebraic equations which are difficult to solve. To overcome this difficulty, a fictitious section was added on top of the pile, and an arbitrary value of Ø assumed at the top of this fictitious section. Using central differences then produced an algorithm that yielded the value of Ø at the next node. This procedure was repeated until the value of Ø at the bottom has been obtained. Depending upon the sign of this, the value of Ø at the top of the fictitious section was iteratively adjusted until a value of Ø close enough to zero was reached at the bottom.

ıce the distribution of forces along the pile is ıown, the corresponding distribution of sidewall shear ;resses is easily obtained, enabling the calculation ' settlements from Eq. (3). An important precaution, ıough, is not to base the settlement calculation on ıe shear stress on the top section of the pile: At the ›p, the shear stress can be very close to the yield ;ress m, and this can introduce a large numerical ·ror. Instead, the settlement was computed from the ıear at the bottom. The shortening of the pile, calcuıted by integrating s' from Eq. (7), was then added to .ve the settlement of the top.

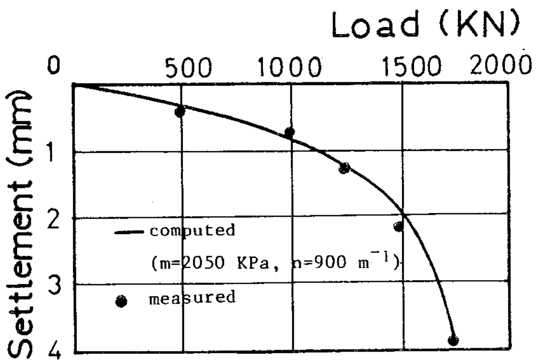
' the procedure described above, The load-settlement ›havior for piles of different diameters and lengths ꜝ predicted, as well as the distribution of axial ›rce for any given load.

:RIVATION OF THE m AND n PARAMETERS

ıe m and n parameters in Eq. (3) can be derived ·om laboratory shear tests between concrete and repreꜝntative rock samples, or preferrably from field tests ꜝrformed on short sockets with a uniform stress dis·ibution. In rock consisting of different strata, m ıd n should be evaluated for each layer separately.

ı homogeneous rock, m and n may also be derived by ıck-calculation from the results of a pile test. Given ıch results, the yield load (and hence the yield shear tress m) can be evaluated by any of the many available ꜝthods (Fellenius 1980). The second parameter (n) can ꜝ back-calculated for any point on the load-settlement ırve by a trial and error technique which may be ›nveniently programmed on a computer. Since each point ill produce a somewhat different n, the final n value an be chosen by the least squares method to provide he best fit. The fit can be further improved by final ɟjustment of the m parameter.

typical example of back-calculation of spring paraeters from load test results is given in Fig. 2. The ile, 300 mm in diameter and 1.2 m long, was drilled in hert and loaded by the embedded piston method (Amir 983a). The computed load-settlement curve, corresponing to the back-calculated values of m = 2050 KPa and = 900 m^{-1}, is shown in Fig. 2 together with the oints obtained from the test.



ig. 2 - Load-settlement curve for pile in chert

COMPARISON WITH EXPERIMENTAL DATA

Caissons in Mica Schist - Philadelphia

A high-rise building in Philadelphia was underpinned on instrumented caissons socketed deep into sound mica schist (Koutsoftas 1981). Since the sockets were designed with a very high factor of safety, the load-settlement curves were essentially linear. Using the yield stress m quoted by the author (1500 KPa), a value of n = 2900 m^{-1} was back-calculated from the load-settlement curve given for a 610 mm diameter socket. The load distribution along the pile was then computed for two different loads, comparing well with the measured values (Fig.3).

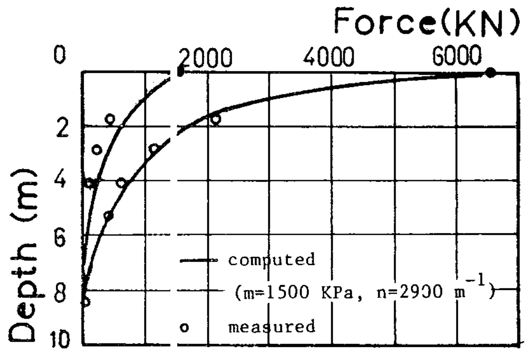


Fig. 3 - Axial force distribution along socket in mica schist

Piles in Mudstone - Melbourne

Four instrumented test piles were installed in moderately weathered mudstone and tested by Williams et al. (1980). Test pile No. M10 (diameter 660 mm, length 7.8 m) settled 3.6 mm under a load of 7660 KN. Based on the back-calculated parameters (m = 900 KPa and n = 2900 m^{-1}), the load distribution along pile M10 was calculated for an applied load of 7660 KN. Again the results (Fig. 4) show a marked resemblance to the measured values.

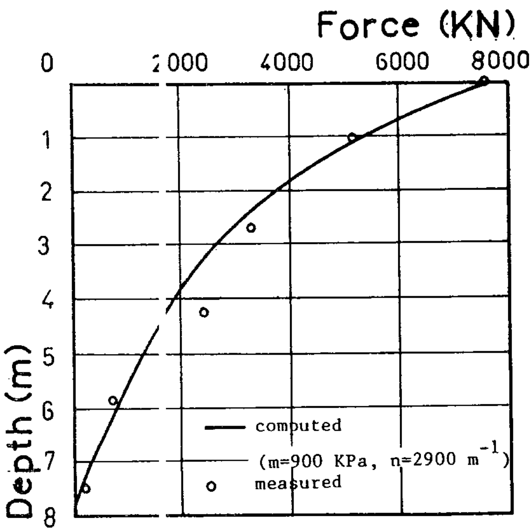


Fig.4 - Axial force distribution along pile in mudstone

CONCLUSIONS

a. The dependence of sidewall shear on displacement may be represented by an exponential function, with the yield stress m and the tangent modulus mn as parameters.

b. Using this function leads to a nonlinear differential equation in terms of force, describing the complete behavior of shear piles in rock.

c. The parameters m and n may be obtained directly from shear tests, either in the field or in the laboratory. In uniform rock, m and n may also be back-calculated from the results of pile load tests.

d. Using the parameters thus obtained, the prediction of load distribution along the pile becomes rather straightforward, and the results show good resemblance to in-situ distributions measured in a variety of rocks.

APPENDIX I - INFLUENCE OF THE DIAMETER ON THE SETTLEMENT OF PILES

According to Mattes & Poulos (1969), the settlement of piles in a linearly elastic half-space is given by:

$$s = \frac{Q}{l\,E_R}\,I_\rho \qquad (16)$$

where I_ρ is an influence factor and E_R the rock mass modulus. For modulus ratios K which are typical of rock, the I_ρ values published by Mattes & Poulos are roughly in direct proportion to the slenderness ratio l/D (Fig. 5). Therefore, I_ρ may be approximately substituted by:

$$I_\rho = B\,\frac{l}{D} \qquad (17)$$

where B = B(K) is constant for a given modulus ratio. Substituting in Eq. (16) results:

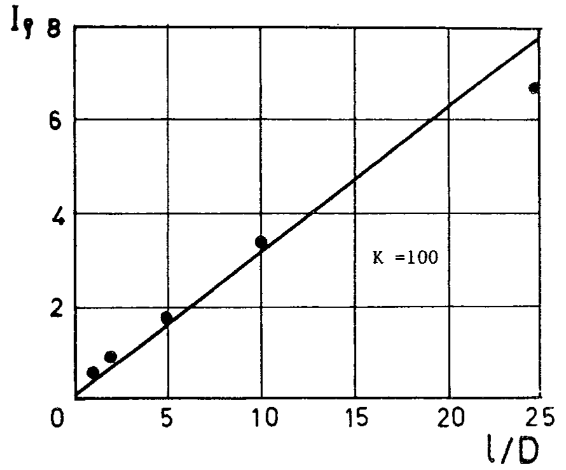


Fig. 5 - Influence factor I_ρ vs. slenderness ratio l/D

$$s = \frac{Q\,B\,l}{l\,E_R\,D} = \frac{\pi\,B\,\bar{F}\,l}{E_R} \qquad (18)$$

where $\bar{F}$ is the mean sidewall shear. The pile modulus M_p is, therefore:

$$M_p = \frac{\bar{F}}{s} = \frac{E_R}{\pi\,B\,l} \qquad (19)$$

From Eq. (19) it emerges that, at least as a first approximation, the pile modulus is independent of the pile diameter D.

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